Shear-Strength Analysis of a Multiring Container

A multiring container is considered which has all rings of the same material, i.e., the same Equation (79b) is assumed valid for all rings, $n = 1, 2, \ldots, N$ with $A_1 = A_2 = \ldots A_N$; $B_1 = B_2 = \ldots B_N$; and $\sigma_1 = \sigma_2 = \ldots = \sigma_N = \sigma$. The pressure-to-strength ratio p_o/σ is derived in exactly the same manner as in Equation (42) (for the specific case $A_n = 3$, $B_n = 2$). The result is

$$\frac{p_o}{\sigma} = \frac{p_N}{\sigma} + \frac{2N}{(A_n + B_n)} \frac{K^{2/N} - 1}{K^{2/N}} - \frac{(A_n - B_n)}{(A_n + B_n)} \frac{q_o - q_N}{\sigma} \tag{81}$$

Similarly, a limit is imposed such that the minimum shear stress, S_{min}, at the bore is greater than or equal to the compressive shear strength of the liner, S_c, i.e.

$$S_{min} \geq - S_c = - \frac{\sigma_c}{2} \tag{82}$$

(This limit is believed to be more realistic than the limit $S_m = 0$ that was used in the earlier analysis.) Using the definition $S_{min} = - S_r + S_m$, the fatigue relation (73b) and the equation for S_r in the liner,

$$S_r = \frac{K^2}{2(K^2 - 1)} \left[(p_o - q_o) - (p_N - q_N) \right] ,$$

in the inequality (82) there results

$$\frac{p_o}{\sigma} \leq \frac{K^2 - 1}{K^2} \frac{B_n}{A_n + B_n} \left(\sigma_c + \frac{2\sigma}{B_n} \right) + (p_n - q_n) + q_o \tag{83}$$

The pressure-to-strength ratio p_o/σ from Equation (82) and the limit (83) are sketched in Figure 63 as functions of p_N, q_N, and q_o. The solid curve for p_o is valid only when it is below the dashed limit curve. The support pressure, p_N, gives the most benefit as shown – both p_o and $(p_o)_{limit}$ increase with p_N. Small amounts of pressure, q_N, are helpful if $p_o \leq (p_o)_{limit}$. A residual bore pressure, q_o, is detrimental – p_o decreases with q_o.

Considering a two-unit, multiring container, it can now be realized that it is best that the fluid support pressure also fluctuates for two reasons:

(1) Too great a residual pressure, q_N, on the inner unit decreases its pressure capability.

(2) The pressure, q_N, on the inner unit corresponds to the pressure, q_o, on the outer unit, which in turn decreases the pressure capability of the outer unit.

The best design in a specific case may not require that $q_N = 0$, but it will require that q_N be sufficiently small.

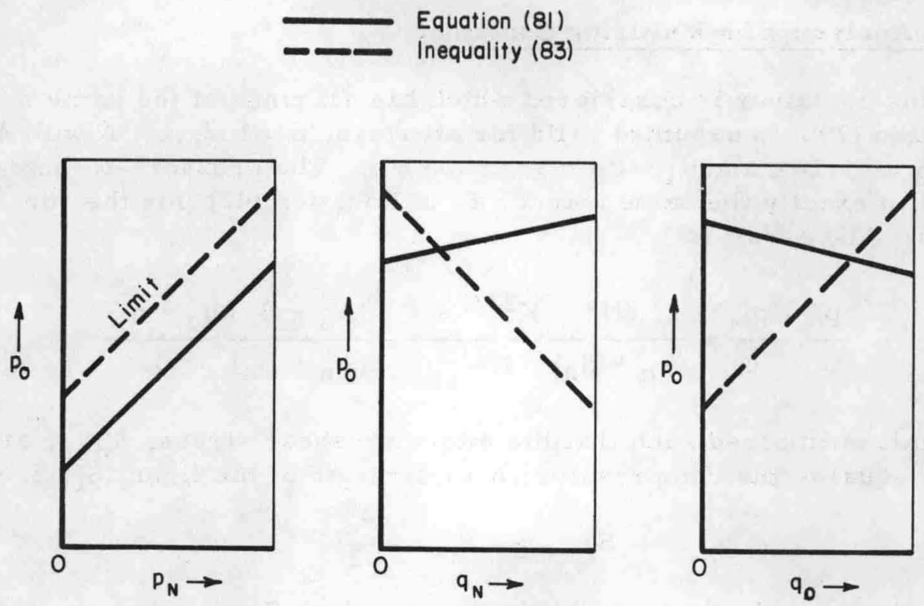

FIGURE 63. INFLUENCE OF PRESSURES p_N, q_N AND q_o ON THE PRESSURE CAPABILITY p_o

Comparison of the Shear and Tensile-Fatigue Criteria

A container designed on the basis of the shear-fatigue criterion will have a predicted pressure capability generally lower than that of a design based upon the tensile fatigue criterion. This is illustrated in Figure 64 for a single-ring (monoblock) container with $p_N = q_o = q_N = 0$. The curves in Figure 64 are plots of the equations

$$p_o/\sigma_u = \frac{2}{(A_n + B_n)} \frac{K^2 - 1}{K^2 + 1} \text{ for the tensile criterion, and} \quad (84)$$

$$p_o/\sigma_u = \frac{2}{(A_n + B_n)} \frac{\sigma_y}{\sigma_u} \frac{K^2 - 1}{K^2} \text{ for the shear criterion} \quad (85)$$

For a large wall ratio (K) the shear criterion predicts lower pressure capability. For thinner walled containers, $K \leq 1.7$, the reverse is true.

For $1.4 \leq K \leq 2.0$ the tensile criterion and the shear criterion both predict about the same pressure capability as shown in Figure 64. This agrees with the conclusion in Reference (46) based upon experimental fatigue data of cylinders with $1.4 \leq K \leq 2.0$ under cyclic internal pressure. However, the shear criterion severely limits the pressure capability for large K. Thick-walled containers, multiring units, are needed to contain the high extrusion pressures and the important question arises, "Which criterion should be used"? The shear criterion curve in Figure 64 is based upon fatigue data from actual pressurized cylinder tests for low-strength ductile steels, having an ultimate tensile strength of $\sigma_u = 126,000$ psi.[35] The tensile criterion curve, however, is based upon rotating-beam and push-pull tests of high-strength steels, $\sigma_u \geq 250,000$ psi. It has been postulated that the tensile criterion holds for the high-strength steel containers under internal pressure. Experimental verification is needed. The successful design of